Natural Language Processing CSCI 4152/6509 — Lecture 16 Efficient Inference with HMM

Instructors: Vlado Keselj
Time and date: 16:05 – 17:25, 26-Oct-2023
Location: Rowe 1011

Previous Lecture

- Reminders: no Lab 7, A2, P1
- **POS tagging: Introduction**
- Reading: [JM] Ch5 Part-of-Speech Tagging
- Open word categories
- Closed word categories
- Other word categories
- **Hidden Markov Model (HMM):**
  - idea, definition, graphical representation
  - HMM assumption

HMM use in POS Tagging

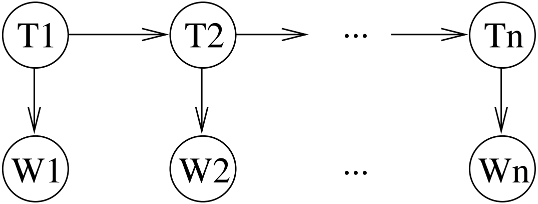
- Hidden states = POS Tags
- Observable variables = words
- In practice: higher-order HMM taggers are used, where the nodes keep a bit longer history (e.g., two previous tags)
- Described in [JM] Sec 5.5 (HMM POS Tagging)

Computational Tasks for HMM

- Evaluation: use HMM assumption formula
- Generation: generate in the order dictated by the "unrolled" graphical representation
- Inference:
  - marginalization, conditioning, completion
  - need for an efficient method (will discuss it)
- Learning: MLE if labeled examples are given

HMM POS Example

- Walk-through example to illustrate inference



- Conditional probability tables required:

$\mathrm{P}(T_1)$, $\mathrm{P}(T_{i+1}|T_i)$, and $\mathrm{P}(W_i|T_i)$

Learning HMM (Training)

- Let us Learn HMM from completely labeled data:

  `swat V flies N like P ants N`

  `time N flies V like P an D arrow N`

- We will use smoothing in word generation, by giving a 0.5 count to all unseen words

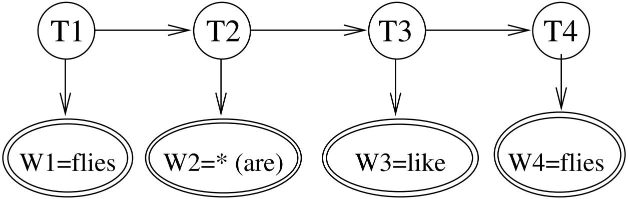
Generated Tables

T_1	$P(T_1)$
N	0.5
V	0.5

T_{i-1}	T_i	$P(T_i \mid T_{i-1})$
D	N	1
N	P	0.5
N	V	0.5
P	D	0.5
P	N	0.5
V	N	0.5
V	P	0.5

T_i	W_i	$P(W_i \mid T_i)$
D	an	$2/3 \approx 0.666666667$
D	*	$1/3 \approx 0.333333333$
N	ants	$2/9 \approx 0.222222222$
N	arrow	$2/9 \approx 0.222222222$
N	flies	$2/9 \approx 0.222222222$
N	time	$2/9 \approx 0.222222222$
N	*	$1/9 \approx 0.111111111$
P	like	0.8
P	*	0.2
V	flies	0.4
V	swat	0.4
V	*	0.2

Tagging Example



$$\arg\max_T \mathrm{P}(T|W = \textsf{sentence}) =$$

$$= \arg\max_T \frac{\mathrm{P}(T, W = \textsf{sentence})}{\mathrm{P}(W = \textsf{sentence})} = \arg\max_T \mathrm{P}(T, W = \textsf{sentence})$$

$$= \arg\max_T \mathrm{P}(T_1) \cdot \mathrm{P}(W_1 = \textsf{flies}|T_1) \cdot \mathrm{P}(T_2|T_1) \cdot \mathrm{P}(W_2 = \textsf{*}|T_2)$$

$$\cdot \mathrm{P}(T_3|T_2) \cdot \mathrm{P}(W_3 = \textsf{like}|T_3) \cdot \mathrm{P}(T_4|T_3) \cdot \mathrm{P}(W_4 = \textsf{flies}|T_4)$$

"Brute-Force" Approach

- Try all combinations of variable values T_1, T_2, T_3, and T_4
- Calculate the overall probability for each of them using the formula

$$\begin{array}{l} \mathrm{P}(T_1) \cdot \mathrm{P}(W_1 = \text{flies}|T_1) \\ \cdot \mathrm{P}(T_2|T_1) \cdot \mathrm{P}(W_2 = *|T_2) \\ \cdot \mathrm{P}(T_3|T_2) \cdot \mathrm{P}(W_3 = \text{like}|T_3) \\ \cdot \mathrm{P}(T_4|T_3) \cdot \mathrm{P}(W_4 = \text{flies}|T_4) \end{array}$$

- Choose the maximal probability

Brute-Force Approach (graphical view)

Efficient Tagging with HMM

- Rather than using the brute-force approach, we can incrementally optimize the product expression by partial maximization from left to right
- One way to represent this is by using a table, which leads to the dynamic programming solution, or the Viterbi algorithm
- The second way to represent this computation is using message passing, or product-sum algorithm

HMM Inference: Dynamic Programming Solution

- Brute-force approach is too inefficient
- Idea for more efficient calculation: maximize sub-products first
- Dynamic Programming approach: divide problem into sub-problems
  - with a manageable number of sub-problems
- Find maximal partial configurations up to T_1, then T_2, T_3, and T_4

Dynamic Programming Approach (graphical view)

Viterbi Algorithm Example

	T_1 (W_1 = flies)	T_2 (W_2 = *)	T_3 (W_3 = like)	T_4 (W_4 = flies)
	$\mathrm{P}(T_1)\mathrm{P}(W_1\|T_1)$	$p \cdot \mathrm{P}(T_2\|T_1)\mathrm{P}(W_2\|T_2)$	$p \cdot \mathrm{P}(T_3\|T_2)\mathrm{P}(W_3\|T_3)$	$p \cdot \mathrm{P}(T_4\|T_3)\mathrm{P}(W_4\|T_4)$
D	$0 \times 0 = 0$	DD: $0 \times 0 \times \frac{1}{3} = 0$ ND: $\frac{1}{9} \times 0 \times \frac{1}{3} = 0$ PD: 0 VD: 0 max: 0	DD: $0 \times 0 \times 0 = 0$ ND: $\frac{1}{90} \times 0 \times = 0$ PD: $\frac{1}{50} \times \frac{1}{2} \times 0 = 0$ VD: $\frac{1}{90} \times 0 \times 0 = 0$ max: 0	DD: $0 \times 0 \times 0 = 0$ ND: $0 \times 0 \times 0 = 0$ PD: $\frac{1}{225} \times 0.5 \times 0 = 0$ VD: $0 \times 0 \times 0 = 0$ max: 0
N	$0.5 \times \frac{2}{9} = \frac{1}{9}$	DN: $0 \times 1 \ldots = 0$ NN: $\frac{1}{9} \times 0 \ldots = 0$ PN: $0 \times \ldots = 0$ VN: $0.2 \times 0.5 \times \frac{1}{9} = \frac{1}{90}$ max: $\frac{1}{90}$	DN: $0 \times 1 \times 0 = 0$ NN: $\frac{1}{90} \times 0 \ldots = 0$ PN: $\frac{1}{50} \times 0.5 \times 0 = 0$ VN: $\frac{1}{90} \times 0.5 \times 0 = 0$ max: 0	DN: $0 \times 1 \times \frac{2}{9} = 0$ NN: $0 \times 0 \times \frac{2}{9} = 0$ PN: $\frac{1}{225} \times 0.5 \times \frac{2}{9} = \frac{1}{2025}$ VN: $0 \times 0.5 \times \frac{2}{9} = 0$ max: $\frac{1}{2025}$
P	$0 \times 0 = 0$	DP: $0 \times \ldots = 0$ NP: $\frac{1}{9} \times 0.5 \times 0.2 = \frac{1}{90}$ PP: $0 \times \ldots = 0$ VP: $0.2 \times 0.5 \times 0.2 = \frac{1}{50}$ max: $\frac{1}{50}$	DP: $0 \times 0 \times 0.8 = 0$ NP: $\frac{1}{90} \times 0.5 \times 0.8 = \frac{1}{225}$ PP: $\frac{1}{50} \times 0 \times 0.8 = 0$ VP: $\frac{1}{90} \times 0.5 \times 0.8 = \frac{1}{225}$ max: $\frac{1}{225}$	DP: $0 \times 0 \times 0 = 0$ NP: $0 \times 0.5 \times 0 = 0$ PP: $\frac{1}{225} \times 0 \times 0 = 0$ VP: $0 \times 0.5 \times 0 = 0$ max: 0
V	$0.5 \times 0.4 = 0.2$	DV: $0 \times \ldots = 0$ NV: $\frac{1}{9} \times 0.5 \times 0.2 = \frac{1}{90}$ PV: $0 \times \ldots = 0$ VV: $0.2 \times 0 \ldots = 0$ max: $\frac{1}{90}$	DV: $0 \times 0 \times 0 = 0$ NV: $\frac{1}{90} \times 0.5 \times 0 = 0$ PV: $\frac{1}{50} \times 0 \times 0 = 0$ VV: $\frac{1}{90} \times 0 \times 0 = 0$ max: 0	DV: $0 \times 0 \times 0.4 = 0$ NV: $0 \times 0.5 \times 0.4 = 0$ PV: $\frac{1}{225} \times 0 \times 0.4 = 0$ VV: $0 \times 0 \times 0.4 = 0$ max: 0