Natural Language Processing CSCI 4152/6509 — Lecture 19 Examples with Message-passing Algorithms

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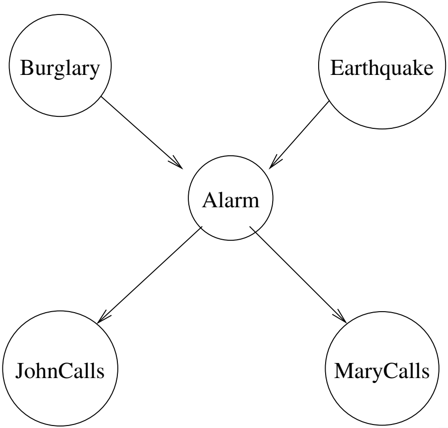
Time and date: 16:05 – 17:25, 7-Nov-2023
Location: Rowe 1011

Previous Lecture

- Message-passing
  1. Isolated factor node to variable node
  2. Isolated variable node to factor node
  3. General factor node to variable node
  4. General variable node to factor node
- Inference tasks using message passing
  1. Marginalization with one variable
  2. Marginalization with multiple variables
  3. Conditioning with one variable
  4. Conditioning with multiple variables
  5. Completion in general

Message Passing Algorithm: Burglar-Earthquake Example

In this example we use the previously given Burglar-Earthquake Bayesian Network:



CP Tables

B	$P(B)$
T	0.001
F	0.999

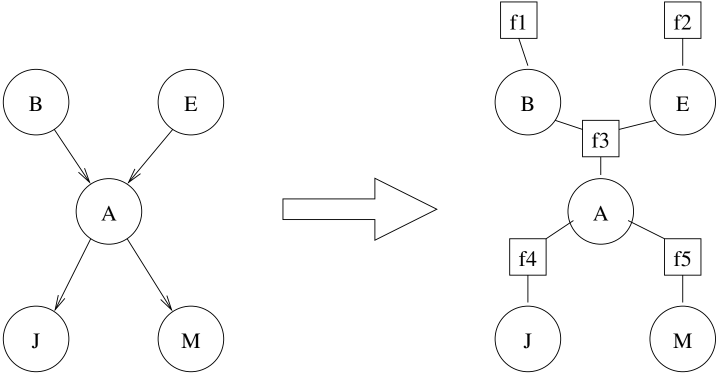
E	$P(E)$
T	0.002
F	0.998

B	E	A	$P(A\|B,E)$
T	T	T	0.95
T	T	F	0.05
T	F	T	0.94
T	F	F	0.06
F	T	T	0.29
F	T	F	0.71
F	F	T	0.001
F	F	F	0.999

A	J	$P(J\|A)$
T	T	0.90
T	F	0.10
F	T	0.05
F	F	0.95

A	M	$P(M\|A)$
T	T	0.70
T	F	0.30
F	T	0.01
F	F	0.99

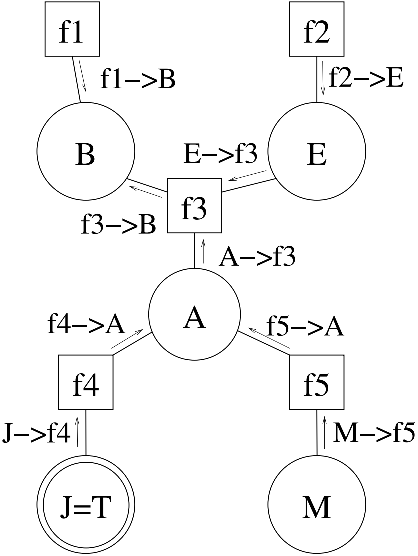
Factor Graph



Burglar-Earthquake Example Problem

- John called, probability that Burglar is in the house
- $P(B = T | J = T) = ?$
- Conditioning with one variable

Message Passing



Message Calculation

B	$f_1 \to B$
T	0.001
F	0.999

E	$f_2 \to E$
T	0.002
F	0.998

E	$E \to f_3$
T	0.002
F	0.998

J	$J \to f_4$
T	1
F	0

M	$M \to f_5$
T	1
F	1

$f_4 \to A$

A	J	$J \to f_4$	f_4	
$A = T$	T	1	$\cdot 0.90$	$= 0.9$
	F	0	$\cdot 0.10$	$= 0$
			Σ	$= 0.9$
$A = F$	T	1	$\cdot 0.05$	$= 0.05$
	F	0	$\cdot 0.95$	$= 0$
			Σ	$= 0.05$

$f_5 \to A$

A	M	$M \to f_5$	f_5	
$A = T$	T	1	$\cdot 0.70$	$= 0.7$
	F	1	$\cdot 0.30$	$= 0.3$
			Σ	$= 1$
$A = F$	T	1	$\cdot 0.01$	$= 0.01$
	F	1	$\cdot 0.99$	$= 0.99$
			Σ	$= 1$

Hence

A	$f_4 \to A$
T	0.9
F	0.05

and

A	$f_5 \to A$
T	1
F	1

. Then:

A	$A \to f_3$
T	0.9
F	0.05

Finally, we compute the message $f_3 \to B$:

$f_3 \to B$						
B	E	A	$E \to f_3$	$A \to f_3$	f_3	
$B = T$	T	T	0.002	$\cdot 0.9$	$\cdot 0.95$	$= 0.00171$
	T	F	0.002	$\cdot 0.05$	$\cdot 0.05$	$= 0.000005$
	F	T	0.998	$\cdot 0.9$	$\cdot 0.94$	$= 0.844308$
	F	F	0.998	$\cdot 0.05$	$\cdot 0.06$	$= 0.002994$
					Σ	$= 0.849017$

$f_3 \to B$						
B	E	A	$E \to f_3$	$A \to f_3$	f_3	
$B=F$	T	T	0.002	$\cdot 0.9$	$\cdot 0.29$	$= 0.000522$
	T	F	0.002	$\cdot 0.05$	$\cdot 0.71$	$= 0.000071$
	F	T	0.998	$\cdot 0.9$	$\cdot 0.001$	$= 0.0008982$
	F	F	0.998	$\cdot 0.05$	$\cdot 0.999$	$= 0.0498501$
					Σ	$= 0.0513413$

Hence, the message $f_3 \to B$ is:

B	$f_3 \to B$
T	0.849017
F	0.0513413

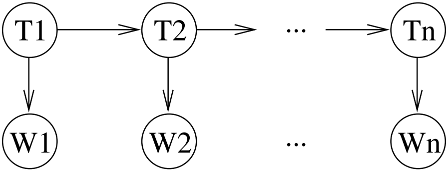
Final Calculation $P(B = T|J = T)$

Now, we can compute $\mathrm{P}(B = T|J = T)$ by multiplying component-wise the messages arriving at B, and by normalizing the result:

$$\begin{aligned} P(B = T|J = T) &= \frac{f_1 \to B(T) \cdot f_3 \to B(T)}{f_1 \to B(T) \cdot f_3 \to B(T) + f_1 \to B(F) \cdot f_3 \to B(F)} \\ &= \frac{0.001 \cdot 0.849017}{0.001 \cdot 0.849017 + 0.999 \cdot 0.513413} = 0.01628373 \end{aligned}$$

Message Passing Algorithm: POS Tagging Example

- HMM Example, revisited



- HMM can be seen as a tree-structured Bayesian Network

Generated Tables

Training data: `swat V flies N like P ants N`
`time N flies V like P an D arrow N`

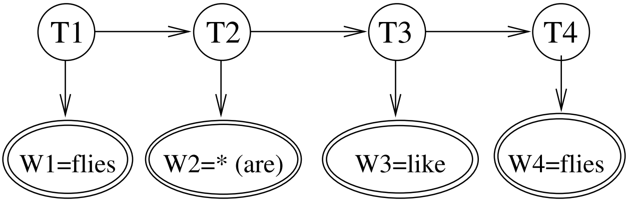
T_1	$P(T_1)$
N	0.5
V	0.5

T_{i-1}	T_i	$P(T_i \mid T_{i-1})$
D	N	1
N	P	0.5
N	V	0.5
P	D	0.5
P	N	0.5
V	N	0.5
V	P	0.5

T_i	W_i	$P(W_i \mid T_i)$
D	an	$2/3 \approx 0.666666667$
D	*	$1/3 \approx 0.333333333$
N	ants	$2/9 \approx 0.222222222$
N	arrow	$2/9 \approx 0.222222222$
N	flies	$2/9 \approx 0.222222222$
N	time	$2/9 \approx 0.222222222$
N	*	$1/9 \approx 0.111111111$
P	like	0.8
P	*	0.2
V	flies	0.4
V	swat	0.4
V	*	0.2

Tagging Example

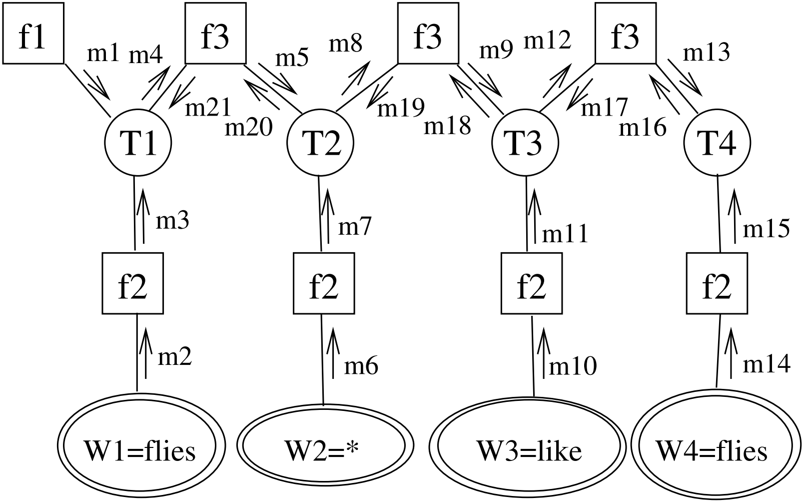
- Example: “flies are like flies”
- Represent HMM as the following Bayesian Network:



POS Tagging as Message Passing

- Solving a completion problem
- Algorithm steps:
  - Create a factor graph
  - Hard-wire output variables
  - Use message passing with maximization
  - Find maximum-likely completion
- We will calculate only necessary messages

Factor Graph (with messages)



T_1	m_1
D	0
N	0.5
P	0
V	0.5

W_1	m_2
flies	1
an	0
*	0
⋮	0

m_3				
$T_1 = D$	$W_1 =$	flies:	$1 \cdot 0$	$= 0$
	$W_1 =$	an:	$0 \cdot \frac{2}{3}$	$= 0$
	$W_1 =$	⋮	⋮	$= 0$
				max:0
$T_1 = N$	$W_1 =$	flies	$: 1 \cdot \frac{2}{9}$	$= \frac{2}{9}$
	$W_1 =$	an	$: 0 \cdot \frac{1}{9}$	$= 0$
				max:$2/9$
⋮				

T_1	m_3
D	0
N	$2/9$
P	0
V	0.4

T_1	$m_4(= m_1 \cdot m_3)$
D	$0 \cdot 0 = 0$
N	$0.5 \cdot 2/9 = 1/9$
P	$0 \cdot 0 = 0$
V	$0.5 \cdot 0.4 = 0.2$

T_2	m_5
D	0
N	0.1
P	0.1
V	$1/18$

m_5	$m_4 \cdot f_3$	
$T_2 = D$	$T_1 = D : 0 \cdot 0$	$= 0$
	$T_1 = N : \frac{1}{9} \cdot 0$	$= 0$
	$T_1 = P : 0 \cdot 0.5$	$= 0$
	$T_1 = V : 0.2 \cdot 0$	$= 0$
		max:0

m_5	$m_4 \cdot f_3$	
$T_2 = N$	$T_1 = D : 0 \cdot 1$	$= 0$
	$T_1 = N : \frac{1}{9} \cdot 0$	$= 0$
	$T_1 = P : 0 \cdot 0.5$	$= 0$
	$T_1 = V : 0.2 \cdot 0.5$	$= 0.1$
		max:0.1

m_5	$m_4 \cdot f_3$	
$T_2 = P$	$T_1 = D : 0 \cdot 0$	$= 0$
	$T_1 = N : \frac{1}{9} \cdot 0.5$	$= 1/18$
	$T_1 = P : 0 \cdot 0$	$= 0$
	$T_1 = V : 0.2 \cdot 0.5$	$= 0.1$
		max:0.1

m_5	$m_4 \cdot f_3$	
$T_2 = V$	$T_1 = D : 0 \cdot 0$	$= 0$
	$T_1 = N : \frac{1}{9} \cdot 0.5$	$= 1/18$
	$T_1 = P : 0 \cdot 0$	$= 0$
	$T_1 = V : 0.2 \cdot 0$	$= 0$
		max:1/18

W_2	m_6
flies	0
an	0
*	1
⋮	0

T_2	m_7
D	$1/3$
N	$1/9$
P	0.2
V	0.2

T_2	$m_8 (= m_5 \cdot m_7)$
D	$0 \cdot \frac{1}{3} = 0$
N	$0.1 \cdot \frac{1}{9} = 1/90$
P	$0.1 \cdot 0.2 = 0.02$
V	$\frac{1}{18} \cdot 0.2 = 1/90$

m_9	$m_8 \cdot f_3$	
$T_3 = D$	$T_2 = D : 0 \cdot 0$	$= 0$
	$T_2 = N : \frac{1}{90} \cdot 0$	$= 0$
	$T_2 = P : \frac{1}{50} \cdot 0.5$	$= 0.01$
	$T_2 = V : \frac{1}{90} \cdot 0$	$= 0$
		max:0.01

m_9	$m_8 \cdot f_3$	
$T_3 = N$	$T_2 = D : 0 \cdot 1$	$= 0$
	$T_2 = N : \frac{1}{90} \cdot 0$	$= 0$
	$T_2 = P : \frac{1}{50} \cdot 0.5$	$= 0.01$
	$T_2 = V : \frac{1}{90} \cdot 0.5$	$= 1/180$
		max:0.01

m_9	$m_8 \cdot f_3$	
$T_3 = P$	$T_2 = D : 0 \cdot 0$	$= 0$
	$T_2 = N : \frac{1}{90} \cdot 0.5$	$= 1/180$
	$T_2 = P : \frac{1}{50} \cdot 0$	$= 0$
	$T_2 = V : \frac{1}{90} \cdot 0.5$	$= 1/180$
		max:1/180

m_9	$m_8 \cdot f_3$	
$T_3 = V$	$T_2 = D : 0 \cdot 0$	$= 0$
	$T_2 = N : \frac{1}{90} \cdot 0.5$	$= 1/180$
	$T_2 = P : \frac{1}{50} \cdot 0$	$= 0$
	$T_2 = V : \frac{1}{90} \cdot 0$	$= 0$
		max:$1/180$

T_3	m_9
D	0.01
N	0.01
P	$1/180$
V	$1/180$

W_3	m_{10}
like	1
$\vdots$	0

T_3	m_{11}
D	0
N	0
P	0.8
V	0

T_3	$m_{12}(= m_9 \cdot m_{11})$
D	$0.01 \cdot 0 = 0$
N	$0.01 \cdot 0 = 0$
P	$\frac{1}{180} \cdot 0.8 = 1/225$
V	$\frac{1}{180} \cdot 0 = 0$

m_{13}	$m_{12} \cdot f_3$	
$T_4 = D$	$T_3 = D : 0 \cdot 0$	$= 0$
	$T_3 = N : 0 \cdot 0$	$= 0$
	$T_3 = P : \frac{1}{225} \cdot 0.5$	$= 1/450$
	$T_3 = V : 0 \cdot 0$	$= 0$
		max:$1/450$

m_{13}	$m_{12} \cdot f_3$	
$T_4 = N$	$T_3 = D : 0 \cdot 1$	$= 0$
	$T_3 = N : 0 \cdot 0$	$= 0$
	$T_3 = P : \frac{1}{225} \cdot 0.5$	$= 1/450$
	$T_3 = V : 0 \cdot 0.5$	$= 0$
		max:$1/450$

m_{13}	$m_{12} \cdot f_3$	
$T_4 = P$	$T_3 = D : 0 \cdot 0$	$= 0$
	$T_3 = N : 0 \cdot 0.5$	$= 0$
	$T_3 = P : \frac{1}{225} \cdot 0$	$= 0$
	$T_3 = V : 0 \cdot 0.5$	$= 0$
		max:0

m_{13}	$m_{12} \cdot f_3$	
$T_4 = V$	$T_3 = D : 0 \cdot 0$	$= 0$
	$T_3 = N : 0 \cdot 0.5$	$= 0$
	$T_3 = P : \frac{1}{225} \cdot 0$	$= 0$
	$T_3 = V : 0 \cdot 0$	$= 0$
		max:0

T_4	m_{13}
D	$1/450$
N	$1/450$
P	0
V	0

W_4	m_{14}
flies	1
$\vdots$	0

T_4	m_{15}
D	0
N	$2/9$
P	0
V	0.4

T_4	$m_{13} \cdot m_{15}$	
D	$\frac{1}{450} \cdot$	$= 0$
N	$\frac{1}{450} \cdot \frac{2}{9}$	$= 1/2025$
P	$0 \cdot 0$	$= 0$
V	$0 \cdot 0.4$	$= 0$

$\boxed{T_4^* = N}$ "hard-wire" T_4

T_4	m_{16}
D	0
N	$2/9$
P	0
V	0

$m_{16} \cdot f_3$	
$\frac{2}{9} \cdot 1$	$= 2/9$
$\frac{2}{9} \cdot 0$	$= 0$
$\frac{2}{9} \cdot 0.5$	$= 1/9$
$\frac{2}{9} \cdot 0.5$	$= 1/9$

T_3	m_{17}
D	$2/9$
N	0
P	$1/9$
V	$1/9$

T_3	$m_9 \cdot m_{11} \cdot m_{17}$	
D	$0.01 \cdot 0 \cdot \frac{2}{9}$	$= 0$
N	$0.01 \cdot 0 \cdot 0$	$= 0$
P	$\frac{1}{180} \cdot 0.8 \cdot \frac{1}{9}$	$= 1/2025$
V	$\frac{1}{180} \cdot 0 \cdot \frac{1}{9}$	$= 0$

$\boxed{T_3^* = P}$

T_3	$m_{18} = m_{17} \cdot m_{11}$
D	0
N	0
P	$\frac{1}{9} \cdot 0.8 = 4/45$
V	0

,

T_2	$m_{19} = m_{18} \cdot f_3$ for $T_3 = P$
D	$\frac{4}{45} \cdot 0 = 0$
N	$\frac{4}{45} \cdot \frac{1}{2} = 2/45$
P	$\frac{4}{45} \cdot 0 = 0$
V	$\frac{4}{45} \cdot \frac{1}{2} = 2/45$

T_2	$m_{19} \cdot m_5 \cdot m_7$	
D	$0 \cdot 0 \cdot \frac{1}{3}$	$= 0$
N	$\frac{2}{45} \cdot 0.1 \cdot \frac{1}{9}$	$= 1/2025$
P	$0 \cdot 0.1 \cdot 0.2$	$= 0$
V	$\frac{2}{45} \cdot \frac{1}{18} \cdot 0.2$	$= 1/2025$

Let us choose $\boxed{T_2^* = V}$.

T_2	$m_{20} = m_7 \cdot m_{19}$
D	0
N	0
P	0
V	$0.2 \cdot \frac{2}{45} = 2/225$

T_1	$m_{21} = m_{20} \cdot f_3$ for $T_2 = V$
D	$\frac{2}{225} \cdot 0 = 0$
N	$\frac{2}{225} \cdot \frac{1}{2} = 1/225$
P	$\frac{2}{225} \cdot 0 = 0$
V	$\frac{2}{225} \cdot 0 = 0$

To find optimal T_1 we calculate:

T_1	$m_1 \cdot m_3 \cdot m_{21}$	
D	$0 \cdot 0 \cdot 0$	$= 0$
N	$0.5 \cdot \frac{2}{9} \cdot \frac{1}{225}$	$= 1/2025$
P	$0 \cdot 0 \cdot 0$	$= 0$
V	$0.5 \cdot 0.4 \cdot 0$	$= 0$

and we obtain $\boxed{T_1^* = N}$.

To summarize, the most probable values of unknown variables T_1, T_2, T_3, and T_4 are:

$$\boxed{T_1^* = N \quad T_2^* = V \quad T_3^* = P \quad T_4^* = N}$$