Natural Language Processing CSCI 4152/6509 — Lecture 17 HMM as Bayesian Network

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Time and date: 16:05 – 17:25, 31-Oct-2022
Location: Rowe 1011

Previous Lecture

- HMM POS example
- HMM Computational tasks
- HMM Brute-force approach
- HMM Inference: Viterbi algorithm

Viterbi Algorithm Example (Repeated)

	T_1 (W_1 = flies)	T_2 (W_2 = *)	T_3 (W_3 = like)	T_4 (W_4 = flies)
	$P(T_1)P(W_1 \mid T_1)$	$p \cdot P(T_2 \mid T_1)P(W_2 \mid T_2)$	$p \cdot P(T_3 \mid T_2)P(W_3 \mid T_3)$	$p \cdot P(T_4 \mid T_3)P(W_4 \mid T_4)$
D	$0 \times 0 = 0$	DD: $0 \times 0 \times \frac{1}{3} = 0$ ND: $\frac{1}{9} \times 0 \times \frac{1}{3} = 0$ PD: 0 VD: 0 max: 0	DD: $0 \times 0 \times 0 = 0$ ND: $\frac{1}{90} \times 0 \times = 0$ PD: $\frac{1}{50} \times \frac{1}{2} \times 0 = 0$ VD: $\frac{1}{90} \times 0 \times 0 = 0$ max: 0	DD: $0 \times 0 \times 0 = 0$ ND: $0 \times 0 \times 0 = 0$ PD: $\frac{1}{225} \times 0.5 \times 0 = 0$ VD: $0 \times 0 \times 0 = 0$ max: 0
N	$0.5 \times \frac{2}{9} = \frac{1}{9}$	DN: $0 \times 1 \ldots = 0$ NN: $\frac{1}{9} \times 0 \ldots = 0$ PN: $0 \times \ldots = 0$ VN: $0.2 \times 0.5 \times \frac{1}{9} = \frac{1}{90}$ max: $\frac{1}{90}$	DN: $0 \times 1 \times 0 = 0$ NN: $\frac{1}{90} \times 0 \ldots = 0$ PN: $\frac{1}{50} \times 0.5 \times 0 = 0$ VN: $\frac{1}{90} \times 0.5 \times 0 = 0$ max: 0	DN: $0 \times 1 \times \frac{2}{9} = 0$ NN: $0 \times 0 \times \frac{2}{9} = 0$ PN: $\frac{1}{225} \times 0.5 \times \frac{2}{9} = \frac{1}{2025}$ VN: $0 \times 0.5 \times \frac{2}{9} = 0$ max: $\frac{1}{2025}$
P	$0 \times 0 = 0$	DP: $0 \times \ldots = 0$ NP: $\frac{1}{9} \times 0.5 \times 0.2 = \frac{1}{90}$ PP: $0 \times \ldots = 0$ VP: $0.2 \times 0.5 \times 0.2 = \frac{1}{50}$ max: $\frac{1}{50}$	DP: $0 \times 0 \times 0.8 = 0$ NP: $\frac{1}{90} \times 0.5 \times 0.8 = \frac{1}{225}$ PP: $\frac{1}{50} \times 0 \times 0.8 = 0$ VP: $\frac{1}{90} \times 0.5 \times 0.8 = \frac{1}{225}$ max: $\frac{1}{225}$	DP: $0 \times 0 \times 0 = 0$ NP: $0 \times 0.5 \times 0 = 0$ PP: $\frac{1}{225} \times 0 \times 0 = 0$ VP: $0 \times 0.5 \times 0 = 0$ max: 0
V	$0.5 \times 0.4 = 0.2$	DV: $0 \times \ldots = 0$ NV: $\frac{1}{9} \times 0.5 \times 0.2 = \frac{1}{90}$ PV: $0 \times \ldots = 0$ VV: $0.2 \times 0 \ldots = 0$ max: $\frac{1}{90}$	DV: $0 \times 0 \times 0 = 0$ NV: $\frac{1}{90} \times 0.5 \times 0 = 0$ PV: $\frac{1}{50} \times 0 \times 0 = 0$ VV: $\frac{1}{90} \times 0 \times 0 = 0$ max: 0	DV: $0 \times 0 \times 0.4 = 0$ NV: $0 \times 0.5 \times 0.4 = 0$ PV: $\frac{1}{225} \times 0 \times 0.4 = 0$ VV: $0 \times 0 \times 0.4 = 0$ max: 0

HMM as Bayesian Network

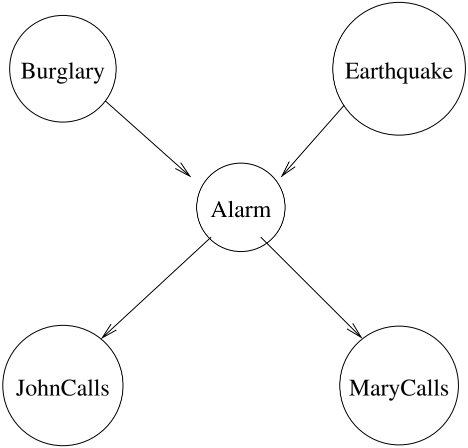
- **Viterbi** algorithm is an **efficient** way to solve a **special** problem:
  - completion with known observables and unknown hidden nodes of an HMM
- **General** approach:
  - Treat HMM as **Bayesian Network**
  - Apply **Product-Sum** (i.e., "Message-passing") algorithm for efficient inference

Bayesian Network Model

- Also known as: Belief Networks, or Bayesian Belief Networks
- A directed acyclic graph (DAG)
  - Each node representing a random variable
  - Edges representing causality (probabilistic meaning)
- Conditional Probability Table (CPT) for each node
- Bayesian Network assumption:

$$\mathrm{P}(\text{ full configuration }) = \prod_{i=1}^{n} \mathrm{P}(V_i | \mathbf{V}_{\pi(i)})$$

Bayesian Network Example



Bayesian Network Assumption

- Bayesian Network Assumption for previous example:

$$\mathrm{P}(B, E, A, J, M) = \mathrm{P}(B)\mathrm{P}(E)\mathrm{P}(A|B, E)\mathrm{P}(J|A)\mathrm{P}(M|A)$$

- Probability of a complete configuration is a product of conditional probabilities
- Each node corresponds to one conditional probability: $\mathrm{P}(B)$, $\mathrm{P}(E)$, $\mathrm{P}(A|B, E)$, $\mathrm{P}(J|A)$, $\mathrm{P}(M|A)$
- CPTs (Conditional Probability Tables are model parameters)

Conditional Probability Tables

B	$\mathrm{P}(B)$
T	0.001
F	0.999

E	$\mathrm{P}(E)$
T	0.002
F	0.998

B	E	A	$\mathrm{P}(A\|B,E)$
T	T	T	0.95
T	T	F	0.05
T	F	T	0.94
T	F	F	0.06
F	T	T	0.29
F	T	F	0.71
F	F	T	0.001
F	F	F	0.999

A	J	$\mathrm{P}(J\|A)$
T	T	0.90
T	F	0.10
F	T	0.05
F	F	0.95

A	M	$\mathrm{P}(M\|A)$
T	T	0.70
T	F	0.30
F	T	0.01
F	F	0.99

Computational Tasks

- Evaluation:

$$P(V_1 = x_1, \ldots, V_n = x_n) = \prod_{i=1}^{n} P(V_i = x_i | \mathbf{V}_{\pi(i)} = \mathbf{x}_{\pi(i)})$$

- Simulation
- Learning from complete observations
- Inference in Bayesian Networks

Inference Example using Brute Force

$$P(B = T | J = T) = \frac{P(B = T, J = T)}{P(J = T)}$$

$$\begin{aligned} P(B = T, J = T) &= \sum_{E,A,M} P(B = T, E, A, J = T, M) \\ &= \sum_{E,A,M} P(B = T)P(E)P(A|B = T, E) \\ &\quad P(J = T|A)P(M|A) \\ &\approx 8.49017 \cdot 10^{-4} \end{aligned}$$

(continued)

$$P(J=T) = P(B=T, J=T) + P(B=F, J=T)$$

$$P(J=T) = P(B=T, J=T) + P(B=F, J=T) \approx$$
$$8.49017 \cdot 10^{-4} + 5.12899587 \cdot 10^{-2} = 0.0521389757$$

$$P(B=T|J=T) = \frac{P(B=T, J=T)}{P(J=T)} \approx$$
$$\frac{8.49017 \cdot 10^{-4}}{0.0521389757} \approx 0.0162837299467699.$$

General Inference in Bayesian Networks

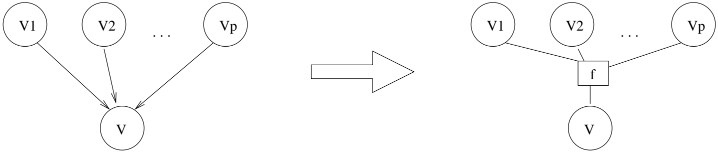
- In some Bayesian Networks inference is always expensive; e.g., joint distribution has a very large number of parameters
- Can we be more efficient if number of parent nodes is limited?
- Naïve Bayes or HMM has a limit of parents to 1
- If we limit number of parents to 2, this may already lead to an NP-hard inference problem
- Proof: a reduction from Circuit Satisfiability problem

Sum-Product Algorithms for Bayesian Networks

- Basic idea: optimizing sum-product calculation using graph structure
  Described in "Factor graphs and the Sum-Product Algorithm" by Kschishang, Frey, and Loeliger in 2000
- Algorithm overview:
  1. Construction of a factor graph
  2. Message-passing algorithms
- Construction of the factor graph
- Principles of message passing

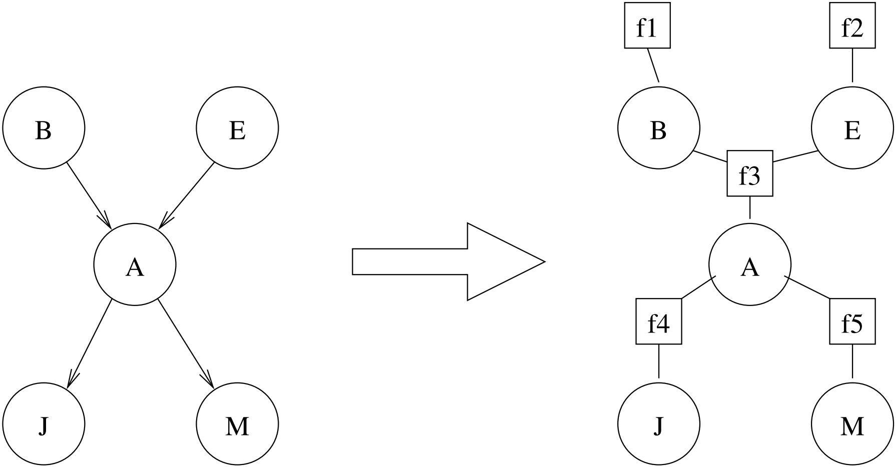
Factor Graph

- Introduce factor nodes:



- Factor graph captures the structure of computation

Factor Graph Example



Principles of Message Passing

- A message summarizes computation in the corresponding part of graph
- Messages are vectors of real numbers
- Each node passes to each neighbour node a message exactly once
- To pass a message to a neighbour node, a node needs to receive messages from all other neighbour nodes
- Important property: a tree-structured Bayesian Network leads to a tree factor graph

Message Passing Ex.: Order of Computation